

# MULTIPLE KALMAN FILTERS IN CONNECTION

N. Assimakis

General Department, National and Kapodistrian University of Athens, Athens, Greece

## ABSTRACT

*The use of multiple Kalman filters in connection is proposed: the first Kalman filter uses actual measurements in order to provide estimation and the other Kalman filters use estimation of the previous Kalman filter in order to provide estimation. It seems that the use of two Kalman filters always improves the estimation. The estimation may improve as the number of Kalman filters increases.*

## KEYWORDS

*Estimation, prediction, Kalman filters, Finite impulse response*

## 1. INTRODUCTION

Estimation plays an important role in many fields of science: applications to aerospace industry, chemical process, communication systems design, control, civil engineering, filtering noise from 2-dimensional images, pollution prediction and power systems are mentioned in [1]. Linear estimation is associated with the following time varying system [1], which consists of the dynamic and the statistical model.

The dynamic model expresses the relationship between state and the measurement and is described by the following state space equations:

$$x(k+1) = F(k+1, k)x(k) + w(k) \quad (1)$$

$$z(k) = H(k)x(k) + v(k) \quad (2)$$

where  $x(k)$  is the  $n \times 1$  state vector,  $z(k)$  is the  $m \times 1$  measurement vector,  $F(k+1, k)$  is the  $n \times n$  transition matrix,  $H(k)$  is the  $m \times n$  output matrix,  $w(k)$  is the  $n \times 1$  state noise and  $v(k)$  is the  $m \times 1$  measurement noise at time  $k \geq 0$ .

The statistical model expresses the nature of state and measurements. The basic underlying assumption is that the state noise,  $w(k)$ , and the measurement noise,  $v(k)$ , follow the white noise distribution:  $\{w(k)\}$  is a zero mean Gaussian process with known covariance  $Q(k)$  of dimension  $n \times n$  and  $\{v(k)\}$  is a zero mean Gaussian process with known covariance  $R(k)$  of dimension  $m \times m$ .

The following assumptions also hold: (a) the initial value of the state  $x(0)$  is a Gaussian random variable with mean  $x_0$  and covariance  $P_0$ , (b) the noise stochastic processes and the random variable  $x(0)$  are independent.

The filtering/estimation problem is to produce an estimate (estimation/prediction) at time  $L$  of the state vector using measurements till time  $L$ . The above is the base for computing the estimation value  $\hat{x}(k/k)$  of the state vector and the corresponding estimation error covariance matrix  $P(k/k)$  as well as the prediction value  $\hat{x}(k+1/k)$  of the state vector and the corresponding prediction error covariance matrix  $P(k+1/k)$ .

The discrete time Kalman filter in [1]-[3] is the most well-known algorithm that solves the filtering problem. Kalman filter is optimal [1]. Kalman filter has been widely used in various applications: Kalman has been used for more accurate estimates of processes affected by temperature fluctuations, such as the power generated by gas turbines [4], the SOC in batteries used for electricity storage [5], or monitoring the temperature profiles inside the heat exchangers serving an ORC unit [6]. A Kalman filter is proposed for the correction of maximum and minimum near surface (2m) temperature forecasts obtained by a Numerical Weather Prediction (NWP) model in [7]. Regional weather forecasts are derived implementing non-linear polynomial functions for different order polynomials using Kalman filters in [8]. Ambient air temperature predictions for “sub-grid” locations are extracted in [9] using Kalman filters. Local terrain topography is used for the adoption of the Kalman Filter technique in [10]. Kalman filtering has been implemented to estimate the electrical load in [11]-[14]. Kalman filter has been used for Global Systems for Mobile (GSM) position tracking in two dimensions [15]. Kalman filter has been used in [16] where the GSM position tracking was derived using models that describe the movement in x-axis and y-axis simultaneously or separately. Also, Mobile Position Tracking in three dimensions using Kalman and Lainiotis filters is presented in [17]. Kalman filter has been used in [18] to obtain level information of tanks available on land and sea vehicles, where one of the biggest problems in measuring the level of tanks is waving and sloshing due to movement. Kalman filter has been designed for estimating the level of a cylindrical tank and thus removing noise from the level sensor [19]. Kalman filter has been implemented to ensure the robustness and reliability of the obtained measurements, due to extreme outdoor working conditions in an oil tanker [20]. Kalman filter has been used for tank level estimation and prediction in [21]-[22]. Kalman filter has been used in tracking the Direction of Arrival (DOA) of a moving source [23]. Kalman filter has been used in motion detection and tracking system for video surveillance [24].

The significance of Kalman filter is undoubtable. This paper deals with the use of multiple Kalman filters in connection. Section 2 summarizes the discrete time Kalman filter algorithms. The proposed estimation algorithm based on the connection of Kalman filters is presented in Section 3. The novelty of the paper concerns the use of Kalman filters in connection in such a way to filter the filtered data. The first Kalman filter uses actual measurements in order to provide estimation and the other Kalman filters use estimation of the previous Kalman filter in order to provide estimation. Each filter uses the same parameters and is optimal. In Section 4 simulation results are presented. Finally, Section 5 summarizes the conclusions.

## 2. KALMAN FILTER

In this section the linear discrete time Kalman filter is summarized for time varying as well for time invariant systems.

### 2.1 Time Varying Kalman Filter

For *time varying systems*, the Time Varying Kalman Filter (TVKF) is derived:

**Time Varying Kalman Filter (TVKF)**

$$K(k)=P(k/k-1)H^T(k)[H(k)P(k/k-1)H^T(k)+R(k)]^{-1} \quad (3)$$

$$x(k/k)=[I-K(k)H(k)]x(k/k-1)+K(k)z(k) \quad (4)$$

$$P(k/k)=[I-K(k)H(k)]P(k/k-1) \quad (5)$$

$$x(k+1/k)=F(k+1,k)x(k/k) \quad (6)$$

$$P(k+1/k)=Q(k)+F(k+1,k)P(k/k)F^T(k+1,k) \quad (7)$$

for  $k=0, 1, \dots$ , with initial conditions  $x(0/-1)=x_0$ ,  $P(0/-1)=P_0$ .

$K(k)$  is the Kalman filter gain.  $I$  denotes the identity matrix.  $M^T$  denotes the transpose of matrix  $M$ . Note that the existence of the inverse of the matrices in (3) is ensured assuming that every covariance matrix  $R(k)$  is positive definite; this has the significance that no measurement is exact.

## 2.2 Time Invariant Kalman Filter

For *time invariant systems* where the system transition matrix, the output matrix, and the noise covariance matrices are constant, the resulting Time Invariant Kalman filter (TIKF) takes the following form:

### Time Invariant Kalman Filter (TIKF)

$$K(k)=P(k/k-1)H^T[HP(k/k-1)H^T+R]^{-1} \quad (8)$$

$$x(k/k)=[I-K(k)H]x(k/k-1)+K(k)z(k) \quad (9)$$

$$P(k/k)=[I-K(k)H]P(k/k-1) \quad (10)$$

$$x(k+1/k)=Fx(k/k) \quad (11)$$

$$P(k+1/k)=Q+FP(k/k)F^T \quad (12)$$

for  $k=0, 1, \dots$ , with initial conditions  $x(0/-1)=x_0$ ,  $P(0/-1)=P_0$ .

## 2.3 Steady State Kalman Filter

For time invariant systems, in [1] it is well known that if the signal process model is asymptotically stable, then there exist steady state values  $P_p$  and  $P_e$  of the prediction and estimation error covariance matrices, respectively. In this case, the resulting discrete time Steady State Kalman Filter (SSKF) filter takes the following form:

### Steady State Kalman Filter (SSKF)

$$x(k/k)=Ax(k-1/k-1)+Bz(k) \quad (13)$$

for  $k=1, 2, \dots$ , with initial condition

$$x(0/0)=[I-K(0)H]x(0/-1)+K(0)z(0) \quad (14)$$

where

$$K(0)=P(0/-1)H^T[HP(0/-1)H^T+R]^{-1} \quad (15)$$

and

$$x(0/-1)=x_0, P(0/-1)=P_0.$$

The coefficients  $A=I-K$  and  $B=K$  are calculated off-line by first solving the corresponding discrete time Riccati equation [1]:

$$P=Q+FPF^T-FPH^T[HPH^T+R]^{-1}HPF^T \quad (16)$$

computing the steady state positive value  $P$  of the prediction error variance and then calculating the steady state gain  $K$ :

$$K=PH^T[HPH^T+R]^{-1} \quad (17)$$

## 2.4 FIR Steady State Kalman Filter

Finally, the Finite Impulse Response (FIR) implementation of the Steady State Kalman Filter is derived as in [25]. In fact, from (13) we can write:

$$x(M+1/M+1)=A^{M+1}x(0/0)+\sum_{i=0}^M A^{M-i}Bz(i+1) \quad (18)$$

In the case of asymptotically stable model, the computed powers of A can be expected to converge to zero [26]. Owing to the computer accuracy, this property of A leads to the conclusion that there exists some M, such that:  $A^M \geq \varepsilon$  and  $A^{M+i} < \varepsilon$ ,  $i=1,2,\dots$ , where  $\varepsilon$  is the convergence criterion. Setting  $A^{M+i}=0$ ,  $i=1,2,\dots$ , we get:

$$x(M+k/M+k)=\sum_{i=0}^M A^{M-i}Bz(i+k) \quad (19)$$

for  $k= 1,2, \dots$

Hence, assuming  $z(k)=0$ ,  $k<0$ , we are able to derive the Finite Impulse Response (FIR) implementation of the Steady State Kalman Filter (FIRSSKF):

### FIR Steady State Kalman Filter (FIRSSKF)

$$x(k/k)=\sum_{i=0}^M C_i z(i+k-M) \quad (20)$$

for  $k= 1,2, \dots$ , with

$$C_i=A^{M-i}B, i=0,1,\dots,M \quad (21)$$

The method requires the knowledge of a subset of previous time measurements to calculate the estimation. The number of the previous time measurements required is the FIR length M, which is computed off-line. The FIRSSKF coefficients are also computed off-line. The method can be used in order to derive estimation at a specific time without computing any previous estimation.

## 3. KALMAN FILTERS IN CONNECTION

The basic idea is to use Kalman filters in connection: the first Kalman filter uses actual measurements in order to provide estimation and the other Kalman filters use estimation of the previous Kalman filter in order to provide estimation. The estimation of the previous Kalman filter is multiplied by the output matrix. Each Kalman filter uses the same parameters. In general we are able to use L Kalman filters, deriving Kalman Filters in connection (KFC) filtering:

### Kalman Filters in connection (KFC)

Kalman Filter 1

input  $z(k)$

output  $x_1(k/k)$

Kalman Filter j,  $j=2,\dots,L$

input  $H(k)x_{j-1}(k/k)$

output  $x_j(k/k)$

It is obvious that we are able to use Time Varying Kalman Filter, Time Invariant Kalman Filter, Steady State Kalman Filter or FIR Steady State Kalman Filter. It is obvious that final results are extracted after the implementation of L filters. Thus the calculation burden of connecting L Kalman filters equals L times the calculation burden of a single Kalman filter.

The connection of L Kalman filters is depicted in Figure 1.

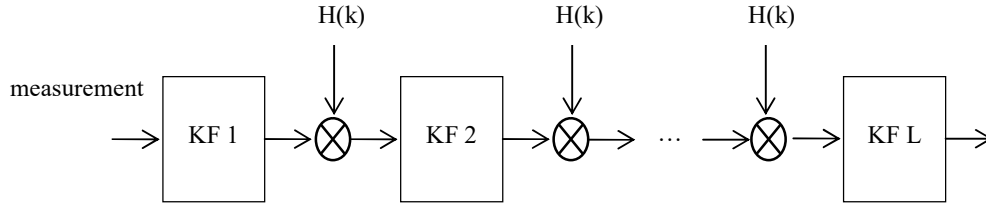


Figure 1. Kalman filters in connection

In the case of the use of FIR Steady State Kalman Filter, all filters have the same coefficients and the same FIR length  $M = M_j, j = 0, 1, \dots, M$ . Denoting the prediction of the  $j$ -th Kalman filter as  $x_j(k/k), j = 1, 2, \dots, L$ , we have:

FIRSSKF 1 with FIR length M

input  $z(k)$

output  $x_1(k/k)$

FIRSSKF  $j, j=2, \dots, L$  with FIR length M

input  $Hx_{j-1}(k/k)$

output  $x_j(k/k)$

Then, we get:

$$x_1(k/k) = \sum_{i=0}^M C_i z(i+k-M) \quad (22)$$

$$x_j(k/k) = \sum_{i=0}^M C_i Hx_{j-1}(i+k-M), j=2, \dots, L \quad (23)$$

It is worth to notice that in the FIR Steady State Kalman Filters case, each filter can be described by its coefficients and hence by its finite impulse response  $h(k)$  derived by the FIRSSKF coefficients. Then we are able to write for each filter:

$$x_1(k/k) = h(k) * z(k) \quad (24)$$

$$x_j(k/k) = h(k) * Hx_{j-1}(k/k), j=2, \dots, L \quad (25)$$

where  $*$  denotes the linear convolution.

This scheme is depicted in Figure 2.

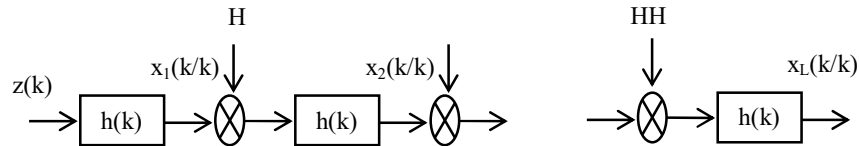


Fig. 2. FIR Steady State Kalman filters in connection

Thus we derived an iterative algorithm for computing the output of each FIR Steady State Kalman Filter.

The final output of the  $L$ -th filter can be obtained by:

$$x_L(k/k) = H^{L-1} \{h(k) * h(k) * \dots * h(k)\} * z(k/k) \quad (26)$$

## 4. SIMULATION RESULTS

### 4.1 Example 1. Constant estimation

This example is taken from [27]. Constant estimation can be implemented using Kalman filter. The Kalman filter parameters are:  $F=1$ ,  $H=1$ ,  $Q=0.00001$ ,  $R=0.01$ . TIKF was implemented with initial conditions  $x_0=0.3$ ,  $P_0=0.01$ . The results for constant estimation, using TIKF are depicted in Figure3. The actual state (constant), and the Kalman filter estimations for  $L=1\dots4$  are plotted for 100 time moments.

The filters perform well and present similar discrepancies with respect to the actual state. They follow the general trend of the actual curve. The estimation becomes better when the number of Kalman filters increases.

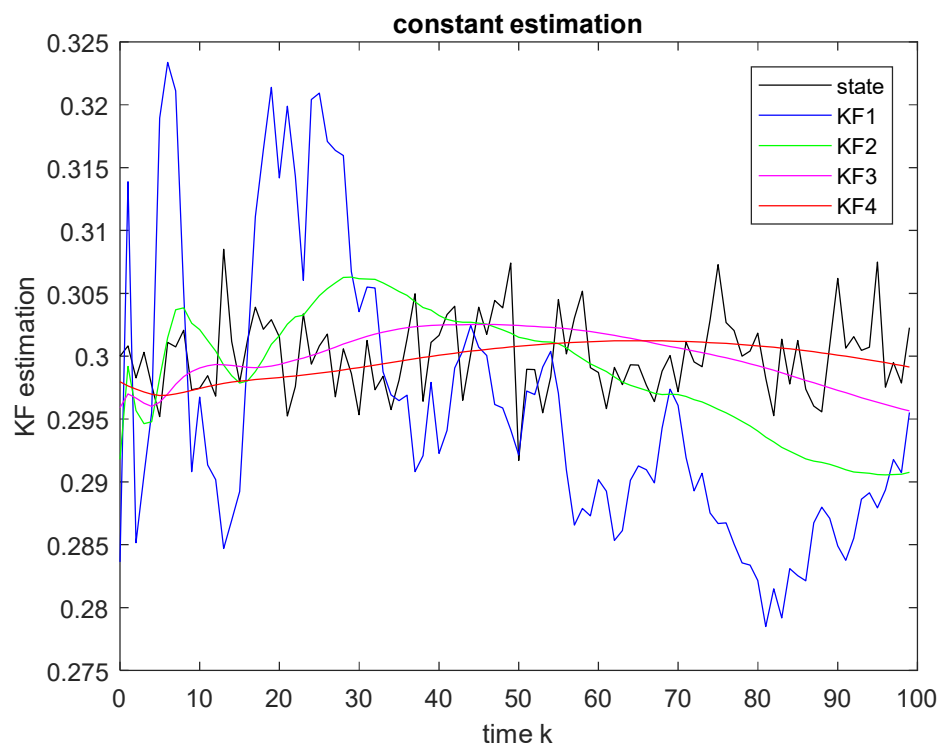


Figure3.Constant estimation using TIKF

In order to measure of the efficiency of the prediction algorithms, the following metrics were computed:

- the mean absolute estimation error of the Kalman Filter estimation with respect to the actual state
- the percent absolute estimation error of the Kalman Filter estimation with respect to the actual state

Table 1 summarizes the mean absolute estimation error and the percent absolute estimation error for Example 1. From Table 1 it is clear that all algorithms provide estimations of high accuracy. It was found that:

1. Estimation becomes better when the number of Kalman filters increases.
2. The use of a small number of Kalman filters leads to negligible improvement in estimation.

Table 1. Example 1. Absolute Mean Error And Percent Absolute Estimation Error

Number of filters	Mean absolute estimation error	% absolute estimation error
1	0.0107	3.5446
2	0.0047	1.5818
3	0.0030	1.0070
4	0.0027	0.8960

#### 4.2 Example 2. Electric load estimation.

This example is taken from [13]. Electric load estimation can be implemented using Kalman filter. The Kalman filter parameters are:  $F=1$ ,  $H=1$ ,  $Q=0.235$ ,  $R=655$ . TIKF was implemented with initial conditions  $x_0=1000$ ,  $P_0=1$ . The results for electric load estimation, using TIKF are depicted in Figure 4. The actual state (electric load), and the Kalman filter estimations for  $L=1..4$  are plotted for 100 time moments.

The filters perform well and present similar discrepancies with respect to the actual state. They follow the general trend of the actual curve. The estimation becomes better when the number of Kalman filters increases.

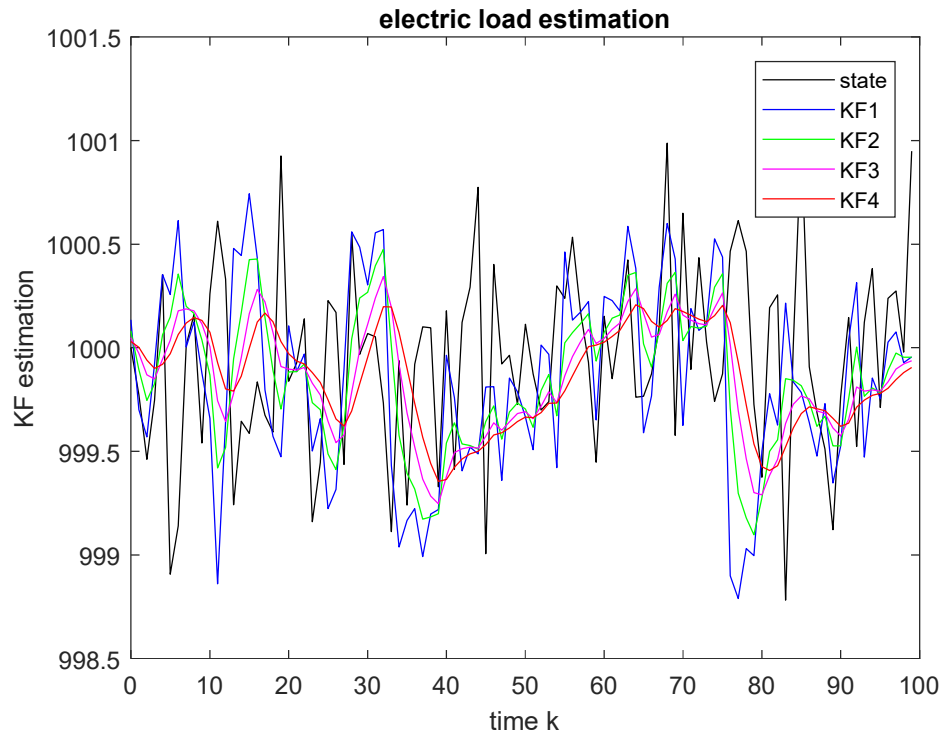


Figure4. Electric load estimation using TIKF

In order to measure of the efficiency of the prediction algorithms, the following metrics were computed:

- the mean absolute estimation error of the Kalman Filter estimation with respect to the actual state
- the percent absolute estimation error of the Kalman Filter estimation with respect to the actual state

Table 2 summarizes the mean absolute estimation error and the percent absolute estimation error for Example 2. From Table 2 it is clear that all algorithms provide estimations of high accuracy. It was found that:

1. Estimation becomes better when the number of Kalman filters increases.
2. The use of a small number of Kalman filters leads to negligible improvement in estimation.

Table 2. Example 2. Absolute Mean Error And Percent Absolute Estimation Error

Number of filters	Mean absolute estimation error	% absolute estimation error
1	0.5095	0.0509
2	0.4597	0.0460
3	0.4300	0.0430
4	0.4128	0.0413

### 4.3 Example 3. Randomwalk estimation.

This example is taken from [28]. Random walk estimation can be implemented using Kalman filter. The Kalman filter parameters are:  $F=1$ ,  $H=1$ ,  $Q=1$ ,  $R=1$ . TIKF was implemented with initial conditions  $x_0=10$ ,  $P_0=0.1$ . The Kalman filter estimations for  $L=1 \dots 20$  were computed.

In order to measure of the efficiency of the prediction algorithms, the following metrics were computed:

- the mean absolute estimation error of the Kalman Filter estimation with respect to the actual state
- the percent absolute estimation error of the Kalman Filter estimation with respect to the actual state

Table 3 summarizes the mean absolute estimation error and the percent absolute estimation error for Example 3. From Table 3 it is clear that all algorithms provide estimations of high accuracy. It was found that:

1. The use of two Kalman filters improves the estimation.
2. The estimation may improve as the number of Kalman filters increases till  $L=10$ . Then the estimation is getting worse.
3. The use of a small number of Kalman filters leads to negligible improvement in estimation.



Table 3. Example 3. Absolute Mean Error And Percent Absolute Estimation Error

Number of filters	Mean absolute estimation error	% absolute estimation error
1	0.8245	8.5024
2	0.7717	7.9646
3	0.7527	7.7653
4	0.7415	7.6453
5	0.7332	7.5566
6	0.7295	7.5149
7	0.7286	7.5011
8	0.7279	7.4900
9	0.7276	7.4829
10	0.7275	7.4794
11	0.7285	7.4885
12	0.7300	7.5026
13	0.7309	7.5120
14	0.7319	7.5221
15	0.7327	7.5304
16	0.7331	7.5352
17	0.7332	7.5377
18	0.7337	7.5432
19	0.7338	7.5449
20	0.7338	7.5457

## 5. CONCLUSIONS

Estimation plays a significant role in many fields of science. Kalman filter is the most well-known estimation algorithm and has been widely and successfully used in various applications. The use of multiple Kalman filters in connection was proposed. The first Kalman filter uses actual measurements in order to provide estimations. The other Kalman filters use estimation of the previous Kalman filter in order to provide estimations.

Time Varying, Time Invariant and Steady State Kalman Filters can be used; filters of the same type are connected. An FIR implementation of the Steady State Kalman Filters was derived. In this case, all filters have same FIR length and the same coefficients.

Connecting Kalman filters, we derived estimation algorithms with the following drawbacks:

1. Final estimations are extracted after the implementation of L filters.
2. The complexity of the algorithm resulting by connecting L Kalman filters is the complexity of the Kalman filter multiplied by L.
3. The estimation may improve as the number of Kalman filters increases, but may also get worse as the number of Kalman filters increases.

Connecting Kalman filters, we derived estimation algorithms with the following advantages:

1. It seems that the use of two Kalman filters always improves the estimation.
2. The estimation may improve as the number of Kalman filters increases.
3. The use of a small number of Kalman filters leads to negligible improvement in estimation.

It is evident that the idea of using Kalman filters in connection, as proposed in this work, can be applied a) in other types of Kalman Filter, such as Extended Kalman Filter (EKF) and b) in other types of filters, such as and Information Filter and Lainiotis filters. Finally, future work

may deal with Kalman filters in connection aiming to derive prediction algorithms, expecting prediction improvement in comparison to the use of a single Kalman filter (which provides estimations as well as predictions).

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#### **Author**

Professor Nicholas Assimakis received the Diploma and Ph.D. degree from the Department of Computer Engineering and Informatics, University of Patras, Greece. He is currently a Professor with the General Department of the National and Kapodistrian University of Athens, Greece. His research interests are in digital signal processing, estimation theory, filtering, algorithm design and development. He has 49 scientific publications in international journals and 24 scientific publications in international conferences. He authored 4 books and co-authored 7 books.

