International Journal of Computer Science and Information Technology (IJCSIT) Vol.1, No.5 March 2021.

DISCRETE RICCATI EQUATION SOLUTION VIA QUADRATIC MATRIX EQUATION

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ABSTRACT

In this note it is shown that the discrete Riccati equation solution can be derived by solving a quadratic matrix equation with well-defined coefficients. This is feasible if the noise covariance matrices and the transition matrix are nonsingular.

KEYWORDS

Discrete Riccati equation,Quadratic matrix equation

THE DISCRETE RICCATI EQUATION AS A QUADRATIC MATRIX EQUATION

The discrete Riccati equation, which arises in linear estimation and is associated with discrete time state space systems, plays a fundamental role in estimation theory [1]. The discrete Riccati equation results from the discrete Kalman filter equations.

Consider the known real matrices $F \in \mathbb{R}^{n\times n}, H \in \mathbb{R}^{m\times n}, Q \in \mathbb{R}^{n\times n}, R \in \mathbb{R}^{m\times m}$, where F is the transition matrix, His the output matrix and the state and measurement noise covariance matricesQ and R are symmetric and nonnegative definite matrices. The discrete Riccati equation:

$$P = Q + F \cdot P \cdot F^T - F \cdot P \cdot H^T \cdot \left(H \cdot P \cdot H^T + R\right)^{-1} \cdot H \cdot P \cdot F^T \qquad (1)$$

has a unique solution $P \in \mathbb{R}^{n\times n}$, which is a real square symmetric nonnegative definite matrix, when well-defined conditions [1] are satisfied. P is the steady state prediction error covariance matrix. Iterative as well as algebraic non-recursive solutions can be found in the literature [1].

Assuming that the measurement noise covariance matrix R is positive definite, equation (1) can be written as:

$$P = Q + F \cdot \left(P^{-1} + H^T \cdot R^{-1} \cdot H\right)^{-1} \cdot F^T \qquad (2)$$

Assuming that the state noise covariance matrix Q is positive definite and using the matrix inversion lemma, equation (2) is written as:

$$P^{-1} = Q^{-1} - Q^{-1} \cdot F \cdot [P^{-1} + H^T \cdot R^{-1} \cdot H + F^T \cdot Q^{-1} \cdot F]^{-1} \cdot F^T \cdot Q^{-1} \qquad (3)$$

The nonsingularity of Q and Rensures the nonsingularity of P, which then it is positive definite.

DOI:10.3121/IJCSiT.2020.1501

Then, adding the quantity $H^T \cdot R^{-1} \cdot H + F^T \cdot Q^{-1} \cdot F$ to both sides of equation (3) and defining:

$$\Pi = P^{-1} + H^T \cdot R^{-1} \cdot H + F^T \cdot Q^{-1} \cdot F \quad (4)$$

$$c = Q^{-1} + H^T \cdot R^{-1} \cdot H + F^T \cdot Q^{-1} \cdot F \quad (5)$$

$$a = Q^{-1} \cdot F \quad (6)$$

it is clear that (3) is written as:

$$\Pi = c - a \cdot \Pi^{-1} \cdot a^T \quad (7)$$

Assuming that the transition matrix F is nonsingular, from (7) results

$$\Pi \cdot a^{-T} \cdot \Pi \cdot a^{-T} = c \cdot a^{-T} \cdot \Pi \cdot a^{-T} - a \cdot a^{-T} \quad (8)$$

Thendefining:

$$X = \Pi \cdot a^{-T} \quad (9)$$

$$B = -c \cdot a^{-T} \quad (10)$$

$$C = a \cdot a^{-T} \quad (11)$$

equation (8) is written as:

$$X^2 + B \cdot X + C = 0 \quad (12)$$

which is a quadratic matrix equation with well-defined coefficients and can be solved using known algorithms [2].

Finally, the solution of the discrete Riccati equation can be derived via the solution of the quadratic matrix equation (12), since using (4) and (9) results

$$P = \left(X \cdot a^T - H^T \cdot R^{-1} \cdot H - F^T \cdot Q^{-1} \cdot F\right)^{-1} \quad (13)$$

In conclusion, if the noise covariance matrices and the transition matrix are nonsingular, then the unique positive definite solution of the discrete Riccati equation can be derived by solving a quadratic matrix equation with well-defined coefficients.

REFERENCES

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[2] N. J. Higham and H.-M.Kim, “Numerical analysis of a quadratic matrix equation,”*IMA Journal of Numerical Analysis*, vol. 20, pp. 499–519, 2000.

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